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Propagator of the Massive vector Boson.

Vector field Z^α の自由ラグランジアン密度

$$\mathcal{L}_Z = -\frac{1}{4} G_{\alpha\beta} G^{\alpha\beta} + \frac{1}{2} M^2 Z_\alpha Z^\alpha$$

$$\frac{\partial \mathcal{L}_Z}{\partial Z^\mu} = +M^2 Z_\mu \quad , \quad \frac{\partial \mathcal{L}_Z}{\partial(\partial^\nu Z^\mu)} = -\frac{1}{2} G_{\alpha\beta} \cdot \frac{\partial G^{\alpha\beta}}{\partial(\partial^\nu Z^\mu)} = -\frac{1}{2} G_{\alpha\beta} \left(g^\alpha{}_\nu g^\beta{}_\mu - g^\beta{}_\nu g^\alpha{}_\mu \right)$$

$$= -\frac{1}{2} \left(G_{\nu\mu} - G_{\mu\nu} \right) = G_{\mu\nu}$$

より、ラグランジュ方程式は

$$\frac{\partial \mathcal{L}_Z}{\partial Z^\mu} - \partial^\nu \frac{\partial \mathcal{L}_Z}{\partial(\partial^\nu Z^\mu)} = +M^2 Z_\mu - \partial^\nu G_{\mu\nu} = 0.$$

$$\Leftrightarrow \partial_\nu (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \cancel{=} M^2 Z^\mu = 0$$

$$Z^\alpha(x) = \sum_{\mathbf{k},\lambda} \frac{1}{\sqrt{2\omega_k V}} \varepsilon^\alpha_{\mathbf{k},\lambda} \left[C_{\mathbf{k},\lambda} e^{-ikx} + C^\dagger_{\mathbf{k},\lambda} e^{ikx} \right] \quad \text{より.}$$

$$\sum_{\mathbf{k}\lambda} \frac{1}{\sqrt{2\omega_k V}} \left[k^2 \varepsilon^\alpha_{\mathbf{k},\lambda} - (k_\beta \cdot \varepsilon^\beta_{\mathbf{k},\lambda}) k^\alpha \cancel{=} - M^2 \varepsilon^\alpha_{\mathbf{k},\lambda} \right] \left(C_{\mathbf{k},\lambda} e^{-ikx} + C^\dagger_{\mathbf{k},\lambda} e^{ikx} \right) = 0$$

$$\therefore (k^2 - M^2) \varepsilon^\alpha_{\mathbf{k},\lambda} - (k_\beta \varepsilon^\beta_{\mathbf{k},\lambda}) k^\alpha = 0.$$

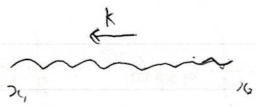
$$\Leftrightarrow \left\{ (k^2 - M^2) g^{\alpha\beta} - k^\alpha k^\beta \right\} \varepsilon_{\beta \mathbf{k},\lambda} = 0$$

Non-zero の偏光が存在するなら. $\det\left[(k^2 - M^2) g^{\alpha\beta} - k^\alpha k^\beta \right] = 0.$

このとき, $\det g^{\alpha\beta} = -1$, $\det(k^\alpha k^\beta) = 0$ より. $k^2 - M^2 = 0$.

これより. $(k_\beta \varepsilon^\beta_{\mathbf{k},\lambda}) k^\alpha = 0$. $\quad \therefore k_\alpha \varepsilon^\alpha_{\mathbf{k},\lambda} = 0$.

$$\langle 0| T\{ Z^\alpha(x_1) Z^\beta(x_2) \} |0\rangle$$



$$= \sum_{\mathbb{k},\lambda} \sum_{\mathbb{k}',\lambda'} \frac{1}{\sqrt{4V^2 \omega_k \omega_{k'}}} \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k}',\lambda'}$$

$$\times \langle 0| T\{ c_{\mathbb{k},\lambda} c_{\mathbb{k}',\lambda'} e^{-ikx_1 - ik'x_2} + \underbrace{c_{\mathbb{k},\lambda} c^\dagger_{\mathbb{k}',\lambda'} e^{-ikx_1 + ik'x_2}}_{\text{ここだけ残る}} + c^\dagger_{\mathbb{k},\lambda} c_{\mathbb{k}',\lambda'} e^{ikx_1 - ik'x_2} + c^\dagger_{\mathbb{k},\lambda} c^\dagger_{\mathbb{k}',\lambda'} e^{ikx_1 + ik'x_2} \} |0\rangle$$

$$= \sum_{\mathbb{k},\lambda} \sum_{\mathbb{k}',\lambda'} \frac{1}{\sqrt{4\omega_k \omega_{k'} V^2}} \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k}',\lambda'} \delta_{\mathbb{k},\mathbb{k}'} \delta_{\lambda\lambda'} T\{ e^{-ikx_1 + ik'x_2} \}$$

$$= \sum_{\mathbb{k},\lambda} \frac{1}{2\omega_k V} \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda} T\{ e^{-i\omega_k (t_1 - t_2) + i\mathbb{k}\cdot(\mathbb{x}_1 - \mathbb{x}_2)} \}$$

連続状態の和に書き換えると、 $\frac{1}{V}\sum_{\mathbb{k}} \to \int \frac{d^3\mathbb{k}}{(2\pi)^3}$ として

$$\int \frac{d^3\mathbb{k}}{(2\pi)^3} \frac{1}{2\omega_k} \left(e^{-i\omega_k t + i\mathbb{k}\cdot\mathbb{x}} \theta(t) + e^{i\omega_k t + i\mathbb{k}\cdot\mathbb{x}} \theta(-t) \right) \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

$$= \int \frac{d^3\mathbb{k}}{(2\pi)^3} \frac{1}{2\omega_k} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left(\frac{e^{+i(\omega-\omega_k)t + i\mathbb{k}\cdot\mathbb{x}}}{\omega - i\epsilon} + \frac{e^{-i(\omega-\omega_k)t + i\mathbb{k}\cdot\mathbb{x}}}{\omega - i\epsilon} \right) \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

$$= \int \frac{d^3\mathbb{k}}{(2\pi)^3} \frac{1}{2\omega_k} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left(\frac{e^{i\omega t + i\mathbb{k}\cdot\mathbb{x}}}{\underbrace{\omega + \omega_k - i\epsilon}_{\omega \to -\omega \text{へ変換}}} + \frac{e^{-i\omega t + i\mathbb{k}\cdot\mathbb{x}}}{\omega + \omega_k - i\epsilon} \right) \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

積分は有限より、変数をシフトしても構わない。

$$= \int \frac{d^3\mathbb{k}}{(2\pi)^3} \frac{1}{2\omega_k} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left(\frac{e^{-i\omega t + i\mathbb{k}\cdot\mathbb{x}}}{\omega + \omega_k - i\epsilon} - \frac{e^{-i\omega t + i\mathbb{k}\cdot\mathbb{x}}}{\omega - \omega_k + i\epsilon} \right) \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

$$= \int \frac{d^3\mathbb{k}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \cdot \frac{-2\omega_k}{2\omega_k} \cdot \frac{e^{-i\omega t + i\mathbb{k}\cdot\mathbb{x}}}{\omega^2 - \omega_k^2 + i\epsilon} \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

$$= i \int \frac{d^3\mathbb{k}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk^0}{2\pi} \frac{e^{-ikx}}{(k^0)^2 - \mathbb{k}^2 - M^2 + i\epsilon} \sum_\lambda \varepsilon^\alpha_{\mathbb{k},\lambda} \varepsilon^\beta_{\mathbb{k},\lambda}$$

内線では、on-shell 条件が使えないため、偏極和について $g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}$ とする必要がある.

$$= i \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ikx}}{k^2 - M^2 + i\epsilon} \left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2} \right) = i D^{\alpha\beta}(x_1 - x_2)$$

$$D^{\alpha\beta}(k) = \frac{g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}}{k^2 - M^2 + i\epsilon}$$