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The Fermion propagator

フェルミオンの伝播関数

$$\langle 0|T\{\psi(x_1)\bar{\psi}(x_2)\}|0\rangle$$

$$=\langle 0|T\left\{\sum_{r,s}\int\frac{d^3p\,d^3q}{(2\pi)^6}\left(a_r(p)u_r(p)e^{-ipx_1}+b_r^+(p)v_r(p)e^{ipx_1}\right)\left(a_s^+(q)\bar{u}_s(q)e^{iqx_2}+b_s(q)\bar{v}_s(q)e^{-iqx_2}\right)\right\}|0\rangle$$

$$=\sum_{r,s}\int\frac{d^3p\,d^3q}{(2\pi)^6}\langle 0|a_r(p)a_s^+(q)|0\rangle u_r(p)\bar{u}_s(q)\,T\{e^{-ipx_1+iqx_2}\}$$

$$=\sum_{r,s}\int\frac{d^3p\,d^3q}{(2\pi)^6}\langle 0|\ (2\pi)^3\delta_{rs}\delta^{(3)}(p-q)-\underbrace{a_s^+(q)a_r(p)|0\rangle}_{0}\ u_r(p)\bar{u}_s(q)\,T\{e^{-ipx_1+iqx_2}\}$$

$$=\sum_{s}\int\frac{d^3p}{(2\pi)^3}u_s(p)\bar{u}_s(p)\,T\{e^{-ip^0(t_1-t_2)}\}e^{ip(x_1-x_2)}$$

$$=\int\frac{d^3p}{(2\pi)^3}\frac{\gamma^\mu p_\mu+m}{2E_p}\left(e^{-iE_p(t_1-t_2)}e^{ip\cdot(x_1-x_2)}\theta(t_1-t_2)+e^{-iE_p(t_2-t_1)}e^{ip\cdot(x_1-x_2)}\theta(t_2-t_1)\right)$$

$$=\int\frac{d^3p}{(2\pi)^3}\frac{\gamma^\mu p_\mu+m}{2E_p}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi i}\left(\frac{-e^{-i(\omega+E_p)(t_1-t_2)+ip\cdot(x_1-x_2)}}{\omega+i\epsilon}+\frac{e^{-i(\omega-E_p)(t_1-t_2)+ip\cdot(x_1-x_2)}}{\omega-i\epsilon}\right)$$

$$=i\int\frac{d^3p}{(2\pi)^3}\frac{\gamma^\mu p_\mu+m}{2E_p}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left(\frac{e^{-i\omega(t_1-t_2)+ip\cdot(x_1-x_2)}}{\omega-E_p+i\epsilon}-\frac{e^{-i\omega(t_1-t_2)+ip\cdot(x_1-x_2)}}{\omega+E_p-i\epsilon}\right)$$

$$=i\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3p}{(2\pi)^3}\frac{(2E_p-2i\epsilon)(\gamma^\mu p_\mu+m)}{\omega^2-E_p^2+2iE_p\epsilon+\epsilon^2}e^{-i\omega(t_1-t_2)+ip\cdot(x_1-x_2)}\frac{1}{2E_p}$$

$$=i\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3p}{(2\pi)^3}\frac{\gamma^\mu p_\mu+m}{\omega^2-p^2-m^2+i\epsilon}e^{-i\omega(t_1-t_2)+ip\cdot(x_1-x_2)}$$

$$=i\int\frac{d^4p}{(2\pi)^4}\frac{\gamma^\mu p_\mu+m}{p^2-m^2+i\epsilon}e^{-ip(x_1-x_2)}\quad = iS_F(x_1-x_2)$$

$$\therefore\ \boxed{S_F(x)=\int\frac{d^4p}{(2\pi)^4}\frac{\not{p}+m}{p^2-m^2+i\epsilon}e^{-ipx}}$$

運動量空間では、 $\boxed{S_F(p)=\dfrac{\not{p}+m}{p^2-m^2+i\epsilon}}$