Fermi 理論から π粒子の崩壊過~~程~~程の相互作用を求める。

Lorentz不変なハミルトニアンとして、

$$\mathcal{H}_{int} = \sum_i (\bar{\Psi}_u \Gamma_i \Psi_d)\left[\bar{\Psi}_e \Gamma_i (C_i^{(1)} + C_i^{(2)}\gamma_5)\Psi_\nu\right] + \text{H.C.}$$ を仮定する。

where $\Gamma_i = \begin{cases} 1 & \text{Scalar} \\ \gamma_\lambda & \text{Vector} \\ \sigma_{\lambda\sigma} & \text{Tensor} \\ i\gamma_5\gamma_\lambda & \text{Axial-Vector} \\ \gamma_5 & \text{Pseudo-Scalar} \end{cases}$、

この相互作用で、以下の崩壊を考える。

$$\pi^- \to \mu + \bar{\nu}_\mu$$
$$\pi^- \to e + \bar{\nu}_e$$

← 今、両方とも相互作用の形が同じで、Field-Strengthが等しいとする。(弱い相互作用の点で共通)

$\pi^- \to e + \bar{\nu}_e$ を考えると、

$$\langle f | \mathcal{H}_{int} | i \rangle = \int d^4x \langle f | \sum_i (\bar{\Psi}_u \Gamma_i \Psi_d)\{\bar{\Psi}_e \Gamma_i (C_i + C_i'\gamma_5)\Psi_{\nu_e}\} | i \rangle$$

$$= \int d^4x \sum_i \left[\left(\frac{1}{\sqrt{V}}\bar{v}_{\bar{u}}(p_{\bar{u}}) e^{-ip_{\bar{u}}x}\right)\Gamma_i\left(\frac{1}{\sqrt{V}} u_d(p_d) e^{-ip_d x}\right)\right]$$
$$\times \left\{\left(\frac{1}{\sqrt{V}}\bar{u}_e(p_e) e^{ip_e x}\right)\Gamma_i (C_i + C_i'\gamma_5)\left(\frac{1}{\sqrt{V}} v_{\bar{\nu}_e}(p_{\bar{\nu}_e}) e^{ip_{\bar{\nu}_e}x}\right)\right\}$$

$$= \frac{1}{V^2}\cdot\sum_i \left(\bar{v}_{\bar{u}}(p_{\bar{u}})\Gamma_i u_d(p_d)\right)\left\{\bar{u}_e(p_e)\Gamma_i (C_i + C_i'\gamma_5) v_{\bar{\nu}_e}(p_{\bar{\nu}_e})\right\}$$
$$\times (2\pi)^4 \delta^{(4)}(p_e + p_{\bar{\nu}_e} - p_{\bar{u}} - p_d)$$

$\sum_i \left(\bar{v}_{\bar{u}}(p_{\bar{u}})\,\Gamma_i\, u_d(p_d)\right)\left\{\bar{u}_e(p_e)\,\Gamma_i\left(C_i + C_i'\gamma_5\right) v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$ について.

スカラー

$\Gamma_i = 1$ のとき、

$$\bar{v}_{\bar{u}}(p_{\bar{u}})\, u_d(p_d)\left\{\bar{u}_e(p_e)\left(C_s + C_s'\gamma_5\right) v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$$

π^- の中では $\bar{u}$ も d も静止しているので、$\mathbb{P}_{\bar{u}} = 0$, $\mathbb{P}_d = 0$ と考える。

すると、

$$v = \begin{pmatrix} 0 \\ h_s \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad u = \begin{pmatrix} h_s \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$u_s(p) = \sqrt{\frac{E_p + m}{2E_p}}\begin{pmatrix} h_s \\ \frac{\sigma\cdot\mathbb{P}}{E_p + m} h_s \end{pmatrix}$$

$$v_s(p) = \sqrt{\frac{|E_p| + m}{2|E_p|}}\begin{pmatrix} -\frac{\sigma\cdot\mathbb{P}}{|E_p| + m} h_s \\ h_s \end{pmatrix}$$

$$h_+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\varphi} \end{pmatrix}$$

$$h_- = \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\varphi} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

として、

$$(0, v_{\bar{u}}^+)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix}$$

$$= \left(0 \ \ -v_{\bar{u}}^+\right)\begin{pmatrix} u_d \\ 0 \end{pmatrix} = 0$$

ベクトル

$\Gamma_i = \gamma_\lambda$ のとき、

$$F = \bar{v}_{\bar{u}}(p_{\bar{u}})\,\gamma_\lambda\, u_d(p_d)\left\{\bar{u}_e(p_e)\,\gamma^\lambda\left(C_V + C_V'\gamma_5\right) v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$$

ここで、

$$\bar{v}_{\bar{u}}(p_{\bar{u}})\,\gamma_0\, u_d(p_d) = (0, v_{\bar{u}}^+)\begin{pmatrix} u_d \\ 0 \end{pmatrix} = 0.$$

$$\bar{v}_{\bar{u}}(p_{\bar{u}})\,\gamma^i\, u_d(p_d) = (0, -v_{\bar{u}}^+)\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix}$$

$$= \left(v_{\bar{u}}^+\sigma^i, \ 0\right)\begin{pmatrix} u_d \\ 0 \end{pmatrix}$$

$$= v_{\bar{u}}^+\sigma^i u_d.$$

よって、$F = -\left(v_{\bar{u}}^+\sigma^i u_d\right)\left\{\bar{u}_e(p_e)\,\gamma^i\left(C_V + C_V'\gamma_5\right) v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$

まとめ、

擬スカラー

$\Gamma_i = \gamma_5$ のとき

$$F = \underbrace{\bar{v}_{\bar{u}}(p_{\bar{u}})\gamma_5 u_d(p_d)}\left\{\bar{u}_e(p_e)\gamma_5\left(C_P + C_P'\gamma_5\right)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$$

$$\shortparallel$$

$$(0, -v^{+}_{\bar{u}})\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}u_d\\ 0\end{pmatrix} = (-v^{+}_{\bar{u}}, 0)\begin{pmatrix}u_d\\ 0\end{pmatrix} = -v^{+}_{\bar{u}}u_d$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$
$$= i\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & \sigma^1\\ -\sigma^1 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^2\\ -\sigma^2 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^3\\ -\sigma^3 & 0\end{pmatrix}$$
$$= i\begin{pmatrix}0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix}\begin{pmatrix}-i\sigma^1 & 0\\ 0 & -i\sigma^1\end{pmatrix}$$
$$= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

よって、

$$F = -(v^{+}_{\bar{u}}u_d)\left\{\bar{u}_e(p_e)\gamma_5\left(C_P + C_P'\gamma_5\right)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$$

テンソル

$$F = \bar{v}_{\bar{u}}(p_{\bar{u}})\sigma_{\lambda\sigma}u_d(p_d)\left\{\bar{u}_e(p_e)\sigma^{\lambda\sigma}\left(C_T + C_T'\gamma_5\right)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$$

- $\sigma_{\lambda\sigma} = \frac{1}{2}(\gamma_\lambda\gamma_\sigma - \gamma_\sigma\gamma_\lambda)$

- $\sigma_{0i} = \frac{1}{2}(\gamma_0\gamma_i - \gamma_i\gamma_0) = \gamma_0\gamma_i = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$

$$= \begin{pmatrix}0 & \sigma^i\\ \sigma^i & 0\end{pmatrix}$$

- $\sigma_{ij} = \frac{1}{2}(\gamma_i\gamma_j - \gamma_j\gamma_i)$

$$= \frac{1}{2}\left\{\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{pmatrix} - \begin{pmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}\right\}$$

$$= \frac{1}{2}\left\{\begin{pmatrix}-\sigma^i\sigma^j & 0\\ 0 & -\sigma^i\sigma^j\end{pmatrix} - \begin{pmatrix}-\sigma^j\sigma^i & 0\\ 0 & -\sigma^j\sigma^i\end{pmatrix}\right\}$$

$$= \frac{-1}{2}\left\{2i\varepsilon^{ijk}\begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix}\right\} = -i\varepsilon^{ijk}\begin{pmatrix}\sigma^k & 0\\ 0 & \sigma^k\end{pmatrix}$$

① $(0, -v_{\bar{u}}^{\dagger})\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix} = (-v_{\bar{u}}^{\dagger}\sigma^i, 0)\begin{pmatrix} u_d \\ 0 \end{pmatrix} = -v_{\bar{u}}^{\dagger}\sigma^i u_d$

② $(0, -v_{\bar{u}}^{\dagger})\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix} = 0$.

$\Rightarrow F = +2\left(v_{\bar{u}}^{\dagger}\sigma^i u_d\right)\left\{\bar{u}_e(p_e)\sigma^{0i}(C_T + C_T'\gamma_5)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$

$\boxed{\text{擬ベクトル}}$

$\bar{v}_{\bar{u}}(p_{\bar{u}})\, i\gamma_5\gamma_\lambda u_d(p_d)\left\{\bar{u}_e(p_e)\, i\gamma_5\gamma^\lambda(C_A + C_A'\gamma_5)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$

↓

$\bar{v}_{\bar{u}}(p_{\bar{u}})\, i\gamma_5\gamma_i u_d(p_d) = -i(0, -v_{\bar{u}}^{\dagger})\begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix}$

$= 0$.

$\bar{v}_{\bar{u}}(p_{\bar{u}})\, i\gamma_5\gamma_0 u_d(p_d) = i(0, -v_{\bar{u}}^{\dagger})\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_d \\ 0 \end{pmatrix}$

$= i(-v_{\bar{u}}^{\dagger}, 0)\begin{pmatrix} u_d \\ 0 \end{pmatrix}$

$= -i v_{\bar{u}}^{\dagger} u_d$

$\gamma_5\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\gamma_5\gamma_i = -\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$

$= -\begin{pmatrix} -\sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$

$= \begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}$

$\therefore F = -i\left(v_{\bar{u}}^{\dagger} u_d\right)\left\{\bar{u}_e(p_e)\, i\gamma_5\gamma^0(C_A + C_A'\gamma_5)v_{\bar{\nu}}(p_{\bar{\nu}})\right\}$

$\boxed{\text{Fermi} \rightarrow \pi\text{崩壊}}$

まとめると、

$$\sum_i \left(\bar{v}_{\bar u}(p_{\bar u})\,\Gamma_i\, u_d(p_d)\right)\left\{\bar{u}_e(p_e)\,\Gamma_i\left(C_i + C_i'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}$$

$$= \left(v_{\bar u}^{\dagger}\sigma^i u_d\right)\left\{\bar{u}_e(p_e)\,\gamma^i\left(C_V + C_V'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}$$

$$- \left(v_{\bar u}^{\dagger} u_d\right)\left\{\bar{u}_e(p_e)\,\gamma_5\left(C_P + C_P'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}$$

$$+ 2\left(v_{\bar u}^{\dagger}\sigma^i u_d\right)\left\{\bar{u}_e(p_e)\,\sigma^{0i}\left(C_T + C_T'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}$$

$$- i\left(v_{\bar u}^{\dagger} u_d\right)\left\{\bar{u}_e(p_e)\, i\gamma_5\gamma^0\left(C_A + C_A'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}$$

$$= -\left(v_{\bar u}^{\dagger} u_d\right)\left[\left\{\bar{u}_e(p_e)\,\gamma_5\left(C_P + C_P'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\} \left\{\bar{u}_e(p_e)\,\gamma_5\gamma_0\left(C_A + C_A'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}\right]$$

$$- \left(v_{\bar u}^{\dagger}\sigma^i u_d\right)\left[\left\{\bar{u}_e(p_e)\,\gamma^i\left(C_V + C_V'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\} - 2\left\{\bar{u}_e(p_e)\,\sigma^{0i}\left(C_T + C_T'\gamma_5\right) v_{\bar\nu}(p_{\bar\nu})\right\}\right]$$