2023/1/14

始状態と終状態で粒子の交換がないダイレクトな項を取り出すと、

$$i\mathcal{M}_{fi} = ie^2 \frac{\bar{u}_{s_1}(q_1)\gamma^\mu u_{r_1}(p_1)\bar{u}_{s_2}(q_2)\gamma_\mu u_{r_2}(p_2)}{(q_1-p_1)^2}$$

$$\mathcal{M}_{fi} = \underbrace{\frac{e^2\bar{u}_{s_1}(q_1)\gamma^0 u_{r_1}(p_1)\bar{u}_{s_2}(q_2)\gamma^0 u_{r_2}(p_2)}{(q_1-p_1)^2}}_{\mathcal{M}_{fi}^{A}} - \frac{e^2\bar{u}_{s_1}(q_1)\gamma^i u_{r_1}(p_1)\bar{u}_{s_2}(q_2)\gamma^i u_{r_2}(p_2)}{(q_1-p_1)^2}$$

固有関数

$$u_s(p) = \sqrt{\frac{E_p+m}{2E_p}}\begin{pmatrix} h_s \\ \frac{\sigma\cdot p}{E_p+m}h_s \end{pmatrix}$$

$$h_+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\varphi} \end{pmatrix}$$

① $\bar{u}_{s_1}(q_1)\gamma^0 u_{r_1}(p_1)$

$$= \sqrt{\frac{E_{q_1}+m}{2E_{q_1}}}\cdot\sqrt{\frac{E_{p_1}+m}{2E_{p_1}}}\left(h_{s_1}^\dagger \quad h_{s_1}^\dagger\frac{\sigma\cdot q_1}{E_{q_1}+m}\right)\begin{pmatrix} h_{r_1} \\ \frac{\sigma\cdot p_1}{E_{p_1}+m}h_{r_1} \end{pmatrix}$$

$$= \left(\frac{E_{q_1}+m}{2E_{q_1}}\right)^{1/2}\left(\frac{E_{p_1}+m}{2E_{p_1}}\right)^{1/2} h_{s_1}^\dagger\left(1+\frac{(\sigma\cdot q_1)(\sigma\cdot p_1)}{(E_{q_1}+m)(E_{p_1}+m)}\right)h_{r_1}$$

$$= \left(\frac{E_{q_1}+m}{2E_{q_1}}\right)^{1/2}\left(\frac{E_{p_1}+m}{2E_{p_1}}\right)^{1/2} h_{s_1}^\dagger\left(1+\frac{q_1\cdot p_1 + i\sigma\cdot(q_1\times p_1)}{(E_{q_1}+m)(E_{p_1}+m)}\right)h_{r_1}$$

② $\bar{u}_{s_2}(q_2)\gamma^0 u_{r_2}(p_2)$

$$= \left(\frac{E_{q_2}+m}{2E_{q_2}}\right)^{1/2}\left(\frac{E_{p_2}+m}{2E_{p_2}}\right)^{1/2} h_{s_2}^\dagger\left(1+\frac{q_2\cdot p_2 + i\sigma\cdot(q_2\times p_2)}{(E_{q_2}+m)(E_{p_2}+m)}\right)h_{r_2}$$

これより、

$$\mathcal{M}_{fi}^{A} = \left(\frac{E_{q_1}+m}{2E_{q_1}}\right)^{1/2}\left(\frac{E_{p_1}+m}{2E_{p_1}}\right)^{1/2}\left(\frac{E_{q_2}+m}{2E_{q_2}}\right)^{1/2}\left(\frac{E_{p_2}+m}{2E_{p_2}}\right)^{1/2}\cdot\frac{1}{(q_1-p_1)^2}$$

$$\times h_{s_1}^\dagger\left(1+\frac{q_1\cdot p_1}{(E_{q_1}+m)(E_{p_1}+m)}+\frac{i\sigma\cdot(q_1\times p_1)}{(E_{q_1}+m)(E_{p_1}+m)}\right)h_{r_1}$$

$$\times h_{s_2}^\dagger\left(1+\frac{q_2\cdot p_2}{(E_{q_2}+m)(E_{p_1}+m)}+\frac{i\sigma\cdot(q_2\times p_2)}{(E_{q_2}+m)(E_{p_2}+m)}\right)h_{r_2}$$

$\boxed{\text{0次の項について}}$

$$\mathcal{M}_0^A = \frac{e^2}{(q_1-p_1)^2}\left(\frac{E_{q_1}+m}{2E_{q_1}}\right)^{1/2}\left(\frac{E_{p_1}+m}{2E_{p_1}}\right)^{1/2}\left(\frac{E_{q_2}+m}{2E_{q_2}}\right)^{1/2}\left(\frac{E_{p_2}+m}{2E_{p_2}}\right)^{1/2}\times(2E_{q_1})\cdot(2E_{q_2})\cdot(2E_{p_1})\cdot(2E_{p_2})$$

$$\times h_{s_1}^{\dagger} h_{r_1} h_{s_2}^{\dagger} h_{r_2}$$

$$= \frac{e^2}{(q_1-p_1)^2}(E_{q_1}+m)^{1/2}(E_{p_1}+m)^{1/2}(E_{q_2}+m)^{1/2}(E_{p_2}+m)^{1/2}\delta_{s_1r_1}\delta_{s_2r_2}$$

$$\Downarrow \quad \text{非相対論的極限} \quad E+m \simeq 2m$$

$$\mathcal{M}_0^A \simeq \frac{e^2}{(q_1-p_1)^2}\cdot(2m)^2\,\delta_{s_1r_1}\delta_{s_2r_2}$$

スピンの保存を表す.

ポテンシャルを求める式

$$\boxed{V(x) = \frac{1}{4m_1m_2}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}\mathcal{M}_{fi} \Rightarrow \frac{1}{4m^2}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}\mathcal{M}_{fi}}$$

よって.

$$V = \cancel{\frac{1}{4m^2}}\int\frac{d^3k}{(2\pi)^3}\cdot\frac{e^2}{k^2}\cdot\cancel{(2m)^2}\,\delta_{s_1r_1}\delta_{s_2r_2}e^{ik\cdot x}$$

$$= \int\frac{d^3k}{(2\pi)^3}\cdot\frac{e^2}{k^2}e^{ik\cdot x}\cdot\delta_{s_1r_1}\delta_{s_2r_2}$$

$$= \frac{e^2}{4\pi|x|}\delta_{s_1r_1}\delta_{s_2r_2}$$