崩壊幅

散乱振幅の2乗は $t, V \to \infty$ で
(遷移)

$$|\langle f|S-1|i\rangle|^2 = (2\pi)^4 \lim_{\substack{t\to\infty \\ V\to\infty}} tV\,\delta(E_f-E_i)\,\delta^{(3)}(\mathbb{P}_f-\mathbb{P}_i)\,|\langle f|\mathcal{T}|i\rangle|^2$$

$\longrightarrow$ 単位時間あたりの遷移確率

$$w = \lim_{\substack{t\to\infty \\ V\to\infty}} V\cdot(2\pi)^4\,\delta(E_f-E_i)\,\delta^{(3)}(\mathbb{P}_f-\mathbb{P}_i)\,|\langle f|\mathcal{T}|i\rangle|^2$$

1粒子 $\to$ n粒子 の場合

$$w(1\to n) = \underbrace{\frac{V}{(2\pi)^3}\int d^3q_1 \cdots \frac{V}{(2\pi)^3}\int d^3q_n}_{n\text{粒子}}\ \overset{\Sigma(\text{状態和})}{\vphantom{|}}|\langle f|\mathcal{T}|i\rangle|^2\cdot(2\pi)^4\,\delta(E_f-E_i)\,\delta^{(3)}(\mathbb{P}_f-\mathbb{P}_i)\,V$$

$$= \frac{V^{n+1}}{(2\pi)^{3n-4}}\int d^3q_1\cdots\int d^3q_n \sum_{state}|\langle f|\mathcal{T}|i\rangle|^2\,\delta(E_f-E_i)\,\delta^{(3)}(\mathbb{P}_f-\mathbb{P}_i)$$

$$= \frac{1}{2E_{\mathbb{P}}(2\pi)^{3n-4}}\int\frac{d^3q_1}{2E_{q_1}}\cdots\int\frac{d^3q_n}{2E_{q_n}}\sum\delta(E_f-E_i)\,\delta(\mathbb{P}_f-\mathbb{P}_i)$$

$$\times V^{n+1}\cdot 2E_{q_1}\cdots 2E_{q_n}\times 2E_{\mathbb{P}}\,|\langle f|\mathcal{T}|i\rangle|^2$$

$$= \frac{(2\pi)^{4-3n}}{2E_P}\int\frac{d^3q_1}{2E_{q_1}}\cdots\int\frac{d^3q_n}{2E_{q_n}}\sum\delta(E_f-E_i)\,\delta(\mathbb{P}_f-\mathbb{P}_i)$$

$$\times V^{n+1}\cdot 2E_{q_1}\cdots 2E_{q_n}\times 2E_P\,|\langle f|\mathcal{T}|i\rangle|^2$$

1 → 2粒子の場合.

$$W(1\to 2) = \frac{1}{2E_P\cdot(2\pi)^2}\int \frac{d^3 q_1}{\cancel{}\; 2E_{q_1}}\cdot\int\frac{d^3 q_2}{\cancel{}\; 2E_{q_2}}\,\delta(E_{q_1}+E_{q_2}-E_P)\,\delta^{(3)}(\mathbf{q}_1+\mathbf{q}_2-\mathbf{P})$$

$$\times V^3\cdot 2E_{q_1}\cdot 2E_{q_2}\cdot 2E_P\,|\langle f|\mathcal{T}|i\rangle|^2$$

積分について.

$$\int\frac{d^3 q_1}{2E_{q_1}}\int\frac{d^3 q_2}{2E_{q_2}}\,\delta(E_{q_1}+E_{q_2}-E_P)\,\delta^{(3)}(\mathbf{q}_1+\mathbf{q}_2-\mathbf{P})$$

$$=\int d^4 q_1\int d^4 q_2\,\delta(q_1^2-M_1^2)\,\theta(q_{10})\,\delta(q_2^2-M_2^2)\,\theta(q_{20})\,\delta^{(4)}(q_1+q_2-P)$$

$$=\int d^4 q_1\,\delta(q_1^2-M_1^2)\,\theta(q_{10})\,\delta([-q_1+P]^2-M_2^2)\,\theta(-q_{10}+E)$$

$q_{10}>0$　　　　$q_{10}<E$

$0<q_{10}<E$

$$=\int_0^E dq_{10}\int_0^\infty d|\mathbf{q}_1|\,|\mathbf{q}_1|^2\int d\Omega\,\delta(q_{10}^2-|\mathbf{q}_1|^2-M_1^2)\,\delta([-q_1+P]^2-M_2^2)$$

$$=\int_0^E dq_{10}\int_0^\infty d|\mathbf{q}_1|\,|\mathbf{q}_1|^2\int d\Omega\,\frac{1}{2|\mathbf{q}_1|}\left\{\underbrace{\delta\left(\sqrt{q_{10}^2-M_1^2}+|\mathbf{q}_1|\right)}_{\text{積分範囲で0}}+\delta\left(\sqrt{q_{10}^2-M_1^2}-|\mathbf{q}_1|\right)\right\}\delta((P-q_1)^2-M_2^2)$$

$$=\int_0^E dq_{10}\int d\Omega\,\frac{\sqrt{q_{10}^2-M_1^2}}{2}\,\delta\left[P^2-2P\cdot q_1+M_1^2-M_2^2\right]$$

重心系をとる.

$\to\ \mathbf{P}=0.$　　$P^2=E^2=s=M^2$

$$= \int_0^{\sqrt{s}} dq_{10} \int d\Omega \, \frac{\sqrt{q_{10}^2 - M_1^2}}{2} \, \delta\left[P^2 - 2P\cdot q_1 + M_1^2 - M_2^2 \right]$$

$\leftarrow \circ \; P^2 = P_0^2 - \underset{\sim}{P}^2 = E^2 = s$, $\underset{\sim}{P} = 0$

$\circ \; P\cdot q_1 = P_0 q_{10} - \underset{\sim}{P}\cdot \underset{\sim}{q}_1 = \sqrt{s}\, q_{10}$

$$= \int_0^{\sqrt{s}} dq_{10} \int d\Omega \, \frac{\sqrt{q_{10}^2 - M_1^2}}{2} \, \delta\left[s - 2\sqrt{s}\, q_{10} + M_1^2 - M_2^2 \right]$$

$$= \int d\Omega \, \frac{1}{2} \sqrt{\frac{(s + M_1^2 - M_2^2)^2}{4s} - M_1^2} \cdot \frac{1}{2\sqrt{s}}$$

$$= \cdots = \int d\Omega \, \frac{\sqrt{\left[s - (M_1 - M_2)^2\right]\left[s - (M_1 + M_2)^2\right]}}{8s}$$

$\Downarrow$ 崩壊幅

$$\Gamma(1 \to 2) = \int d\Omega \, \frac{1}{2E_p \times (2\pi)^2} \cdot \frac{\sqrt{\left[s - (M_1 - M_2)^2\right]\left[s - (M_1 + M_2)^2\right]}}{8s} \left|\langle f | t | i \rangle\right|^2$$

$$= \frac{4\pi}{2M \cdot 4\pi^2} \frac{\sqrt{\left[M^2 - (M_1 - M_2)^2\right]\left[M^2 - (M_1 + M_2)^2\right]}}{8M^2} \left|\langle f | t | i \rangle\right|^2$$

$$= \frac{1}{32\pi M^3} \sqrt{\left[M^2 - (M_1 - M_2)^2\right]\left[M^2 - (M_1 + M_2)^2\right]} \left|\langle f | t | i \rangle\right|^2$$