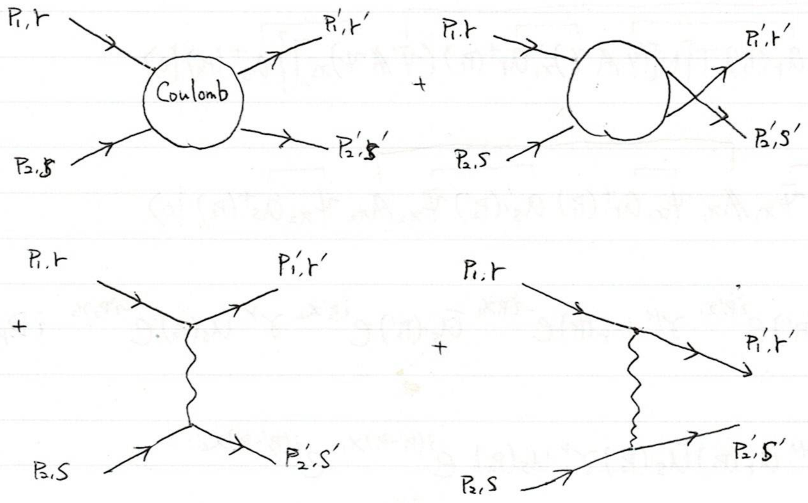
電子-電子散乱

クーロンゲージで計算する場合、



クーロンハミルトニアンは

$$H_{\mathrm{Coul}} = \frac{e^2}{2}\int d^3x_1 \int d^3x_2 \frac{(\bar{\psi}\gamma^0\psi)_{x_1}(\bar{\psi}\gamma^0\psi)_{x_2}}{4\pi|x_1 - x_2|}$$

フォトンの相互作用の2次は

$$S^{(2)} = \frac{(-ie)^2}{2}\int d^4x_1 \int d^4x_2\, T\left\{N\left[(\bar{\psi}\not{A}\psi)_{x_1}(\bar{\psi}\not{A}\psi)_{x_2}\right]\right\}$$

$$T_{fi}^{A} = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0|\, a_{s'}(p_2')\, a_{r'}(p_1')\, T\{N[(\bar\psi A\!\!\!/ \psi)_{x_1} (\bar\psi A\!\!\!/ \psi)_{x_2}]\}\, a_r^\dagger(p_1)\, a_s^\dagger(p_2)|0\rangle$$

$$= -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0|\, a_{s'}(p_2')\, a_{r'}(p_1')\, T\{N[(\bar\psi A\!\!\!/ \psi)_{x_1}\, a_r^\dagger(p_1)\, (\bar\psi A\!\!\!/ \psi)_{x_2}]\}\, a_s^\dagger(p_2)|0\rangle$$

$$= -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0|\, a_{r'}(p_1')\, \bar\psi_{x_1} A\!\!\!/_{x_1} \psi_{x_1}\, a_r^\dagger(p_1)\, a_{s'}(p_2')\, \bar\psi_{x_2} A\!\!\!/_{x_2} \psi_{x_2}\, a_s^\dagger(p_2)|0\rangle$$

$$= -\frac{e^2}{2}\cdot\frac{1}{V^2}\int d^4x_1 \int d^4x_2\, \bar u_{r'}(p_1')\, e^{ip_1'x_1}\, \gamma^\mu\, u_r(p_1)\, e^{-ip_1x_1}\, \bar u_{s'}(p_2')\, e^{ip_2'x_2}\, \gamma^\nu\, u_s(p_2)\, e^{-ip_2x_2}\; iD_{\mu\nu}(x_1-x_2)$$

$$= -\frac{ie^2}{2V^2}\int d^4x_1 \int d^4x_2\, \bar u_{r'}(p_1')\gamma^\mu u_r(p_1)\bar u_{s'}(p_2')\gamma^\nu u_s(p_2)\; e^{i(p_1'-p_1)x_1}\; e^{i(p_2'-p_2)x_2}$$

$$\times \int \frac{d^4k}{(2\pi)^4}\;\frac{G_{\mu\nu}(k)}{k^2-i\epsilon}\; e^{-ik(x_1-x_2)}$$

$$= -\frac{ie^2}{2V^2}\int d^4x_1 \int d^4x_2\, \bar u_{r'}(p_1')\gamma^\mu u_r(p_1)\bar u_{s'}(p_2')\gamma^\nu u_s(p_2)\int \frac{d^4k}{(2\pi)^4}\;\frac{G_{\mu\nu}(k)}{k^2-i\epsilon}\; e^{i(p_1'-p_1-k)x_1}\; e^{i(p_2'-p_2+k)x_2}$$

$$= -\frac{ie^2}{2V^2}\int \frac{d^4k}{(2\pi)^4}\, \bar u_{r'}(p_1')\gamma^\mu u_r(p_1)\bar u_{s'}(p_2')\gamma^\nu u_s(p_2)\;\frac{G_{\mu\nu}(k)}{k^2-i\epsilon}\;(2\pi)^4\delta^{(4)}(p_1'-p_1-k)\times(2\pi)^4\delta^{(4)}(p_2'-p_2+k)$$

$$= -\frac{ie^2}{2V^2}\;\frac{1}{k^2-i\epsilon}\;\underbrace{\bar u_{r'}(p_1')\gamma^\mu u_r(p_1)\bar u_{s'}(p_2')\gamma^\nu u_s(p_2)\, G_{\mu\nu}(k)}_{M^A}\;\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$M^A = \bar u_{r'}(p_1')\gamma^m u_r(p_1)\bar u_{s'}(p_2')\gamma^n u_s(p_2)\left(\delta_{mn}-\frac{k_m k_n}{\mathbb{k}^2}\right)$$

$$= \bar u_{r'}(p_1')\gamma^m u_r(p_1)\bar u_{s'}(p_2')\gamma^m u_s(p_2) - \bar u_{r'}(p_1')\vec\gamma\cdot\hat{\mathbb{k}}\, u_r(p_1)\bar u_{s'}(p_2')\vec\gamma\cdot\hat{\mathbb{k}}\, u_s(p_2)$$

$$T_{fi}^{A} = -\frac{ie^2}{2V^2}\;\frac{1}{k^2-i\epsilon}\left[\{\bar u_{r'}(p_1')\vec\gamma\, u_r(p_1)\}\{\bar u_{s'}(p_2')\vec\gamma\, u_s(p_2)\} - \bar u_{r'}(p_1')\vec\gamma\cdot\hat{\mathbb{k}}\, u_r(p_1)\bar u_{s'}(p_2')\vec\gamma\cdot\hat{\mathbb{k}}\, u_s(p_2)\right]$$

$$\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$T_{fi}^{B} = -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0 | a_{s'}(p_2') a_{r'}(p_1') T\{N[(\bar{\psi} A\!\!\!/ \psi)_{x_1} (\bar{\psi} A\!\!\!/ \psi)_{x_2}]\} a_r^\dagger(p_1) a_s^\dagger(p_2) | 0 \rangle$$

$$= -\frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0 | a_{s'}(p_2') a_{r'}(p_1') \bar{\psi}_{x_1} A\!\!\!/_{x_1} \psi_{x_1} a_r^\dagger(p_1) \bar{\psi}_{x_2} A\!\!\!/_{x_2} \psi_{x_2} a_s^\dagger(p_2) | 0 \rangle$$

$$= \frac{e^2}{2}\int d^4x_1 \int d^4x_2 \langle 0 | a_{s'}(p_2') \bar{\psi}_{x_1} A\!\!\!/_{x_1} \psi_{x_1} a_r^\dagger(p_1) a_{r'}(p_1') \bar{\psi}_{x_2} A\!\!\!/_{x_2} \psi_{x_2} a_s^\dagger(p_2) | 0 \rangle$$

$$= \frac{e^2}{2}\cdot\frac{1}{V^2}\int d^4x_1 \int d^4x_2\, \bar{u}_{s'}(p_2') e^{ip_2'x_1} \gamma^\mu u_r(p_1) e^{-ip_1x_1}\, \bar{u}_{r'}(p_1') e^{ip_1'x_2} \gamma^\nu u_s(p_2) e^{-ip_2x_2}$$

$$\times\, i\int \frac{d^4q}{(2\pi)^4} \frac{G_{\mu\nu}(q)}{q^2 - i\epsilon} e^{-iq(x_1-x_2)}$$

$$= \frac{ie^2}{2V^2}\int d^4x_1 \int d^4x_2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - i\epsilon} \bar{u}_{s'}(p_2') \gamma^\mu u_r(p_1) \bar{u}_{r'}(p_1') \gamma^\nu u_s(p_2)\, G_{\mu\nu}(q)\, e^{i(p_2'-p_1-q)x_1} e^{i(p_1'-p_2+q)x_2}$$

$$= \frac{ie^2}{2V^2}\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - i\epsilon} \bar{u}_{s'}(p_2') \gamma^\mu u_r(p_1) \bar{u}_{r'}(p_1') \gamma^\nu u_s(p_2)\, G_{\mu\nu}(q)\, (2\pi)^4 \delta^{(4)}(p_2'-p_1-q) \times (2\pi)^4 \delta^{(4)}(p_1'-p_2+q)$$

$$= \frac{ie^2}{2V^2} \frac{1}{q^2 - i\epsilon} \bar{u}_{s'}(p_2') \gamma^m u_r(p_1) \bar{u}_{r'}(p_1') \gamma^n u_s(p_2) \left(\delta_{mn} - \frac{q_m q_n}{|\mathbf{q}|^2}\right) \times (2\pi)^4 \delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$= \frac{ie^2}{2V^2}\cdot\frac{1}{q^2 - i\epsilon}\left[\{\bar{u}_{s'}(p_2') \vec{\gamma} u_r(p_1)\}\cdot\{\bar{u}_{r'}(p_1') \vec{\gamma} u_s(p_2)\} - \bar{u}_{s'}(p_2') \vec{\gamma}\cdot\hat{\mathbf{q}}\, u_r(p_1) \bar{u}_{r'}(p_1') \vec{\gamma}\cdot\hat{\mathbf{q}}\, u_s(p_2)\right]$$

$$\times (2\pi)^4 \delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$\therefore T_{fi}^{A+B} = -\frac{ie^2}{2V^2}\Big[$$

$$T_{fi}^{Coul.A} = -i\cdot\frac{e^2}{2}\int dt\int d^3x_1\int d^3x_2\,\frac{1}{4\pi|x_1-x_2|}\langle 0|\,a_{s'}(p_2')\,a_{r'}(p_1')\,(\bar\psi\gamma^0\psi)_{x_1}\,(\bar\psi\gamma^0\psi)_{x_2}\,a_r^\dagger(p_1)\,a_s^\dagger(p_2)\,|0\rangle$$

$$= -\frac{ie^2}{2}\int dt\int d^3x_1\int d^3x_2\,\frac{1}{4\pi|x_1-x_2|}\langle 0|\,a_{r'}(p_1')\,\bar\psi_{x_1}\gamma^0\psi_{x_1}\,a_r^\dagger(p_1)\,a_{s'}(p_2')\,\bar\psi_{x_2}\gamma^0\psi_{x_2}\,a_s^\dagger(p_2)\,|0\rangle$$

$$= -\frac{ie^2}{2V^2}\int dt\int d^3x_1\int d^3x_2\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{|k|^2}\,\bar u_{r'}(p_1')\,e^{iE_{p_1'}t-ip_1'\cdot x_1}\,\gamma^0\,u_r(p_1)\,e^{-iE_{p_1}+ip_1\cdot x_1}$$

$$\times\,\bar u_{s'}(p_2')\,e^{iE_{p_2'}t-ip_2'\cdot x_2}\,\gamma^0\,u_s(p_2)\,e^{-iE_{p_2}t+ip_2\cdot x_2}\,e^{ik\cdot(x_1-x_2)}$$

$$= -\frac{ie^2}{2V^2}\int dt\int d^3x_1\int d^3x_2\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{|k|^2}\,\bar u_{r'}(p_1')\gamma^0u_r(p_1)\,\bar u_{s'}(p_2')\gamma^0u_s(p_2)$$

$$\times\, e^{i(E_{p_1'}+E_{p_2'}-E_{p_1}-E_{p_2})t}\,e^{i(k-p_1'+p_1)\cdot x_1}\,e^{i(-k-p_2'+p_2)\cdot x_2}$$

$$= -\frac{ie^2}{2V^2}\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{|k|^2}\,\bar u_{r'}(p_1')\gamma^0u_r(p_1)\,\bar u_{s'}(p_2')\gamma^0u_s(p_2)\times(2\pi)\,\delta(E_{p_1'}+E_{p_2'}-E_{p_1}-E_{p_2})$$

$$\times\,(2\pi)^3\delta^{(3)}(k-p_1'+p_1)\times(2\pi)^3\delta^{(3)}(-k-p_2'+p_2)$$

$$= -\frac{ie^2}{2V^2}\,\frac{1}{|k|^2}\,\bar u_{r'}(p_1')\gamma^0u_r(p_1)\,\bar u_{s'}(p_2')\gamma^0u_s(p_2)\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_1)$$

$k = p_1' - p_1$

$$T_{p_1'}^{\mathrm{Coul},B} = -i\frac{e^2}{2}\int dt\int d^3x_1\int d^3x_2\,\frac{1}{4\pi|x_1-x_2|}\langle 0|\,a_{s'}(p_2')\,a_{r'}(p_1')\,(\bar\psi\gamma^0\psi)_{x_1}\,(\bar\psi\gamma^0\psi)_{x_2}\,a_r^\dagger(p_1)\,a_s^\dagger(p_2)\,|0\rangle$$

$$= -i\frac{e^2}{2}\int dt\int d^3x_1\int d^3x_2\,\frac{1}{4\pi|x_1-x_2|}\langle 0|\,a_{s'}(p_2')\,\bar\psi_{x_1}\gamma^0\psi_{x_1}\,a_{r'}(p_1')\,\bar\psi_{x_2}\gamma^0\psi_{x_2}\,a_r^\dagger(p_1)\,a_s^\dagger(p_2)\,|0\rangle$$

$$= +i\frac{e^2}{2}\int dt\int d^3x_1\int d^3x_2\,\frac{1}{4\pi|x_1-x_2|}\langle 0|\,a_{s'}(p_2')\,\bar\psi_{x_1}\gamma^0\psi_{x_1}\,a_r^\dagger(p_1)\,a_{r'}(p_1')\,\bar\psi_{x_2}\gamma^0\psi_{x_2}\,a_s^\dagger(p_2)\,|0\rangle$$

$$= \frac{ie^2}{2V^2}\int dt\int d^3x_1\int d^3x_2\int\frac{d^3q}{(2\pi)^3}\frac{1}{|q|^2}\,e^{iq\cdot(x_1-x_2)}\,\bar u_{s'}(p_2')\,e^{iE_{p_2'}t-ip_2'\cdot x_1}\,\gamma^0\,u_r(p_1)\,e^{-iE_{p_1}t+ip_1\cdot x_1}$$

$$\times\,\bar u_{r'}(p_1')\,e^{iE_{p_1'}t-ip_1'\cdot x_2}\,\gamma^0\,u_s(p_2)\,e^{-iE_{p_2}t+ip_2\cdot x_2}$$

$$= \frac{ie^2}{2V^2}\int dt\int d^3x_1\int d^3x_2\int\frac{d^3q}{(2\pi)^3}\frac{1}{|q|^2}\,\bar u_{s'}(p_2')\gamma^0 u_r(p_1)\,\bar u_{r'}(p_1')\gamma^0 u_s(p_2)$$

$$\times\,e^{i(E_{p_1'}+E_{p_2'}-E_{p_1}-E_{p_2})t}\;e^{i(q-p_2'+p_1)\cdot x_1}\;e^{i(-q-p_1'+p_2)\cdot x_2}$$

$$= \frac{ie^2}{2V^2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{|q|^2}\cdot\bar u_{s'}(p_2')\gamma^0 u_r(p_1)\,\bar u_{r'}(p_1')\gamma^0 u_s(p_2)\;\times(2\pi)\,\delta(E_{p_1'}+E_{p_2'}-E_{p_1}-E_{p_2})$$

$$\times\,(2\pi)^3\delta^{(3)}(q-p_2'+p_1)\;\times(2\pi)^3\delta^{(3)}(-q-p_1'+p_2)$$

$$= \frac{ie^2}{2V^2}\,\frac{1}{|q|^2}\,\bar u_{s'}(p_2')\gamma^0 u_r(p_1)\,\bar u_{r'}(p_1')\gamma^0 u_s(p_2)\;\times(2\pi)^4\,\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$q = p_2' - p_1$

$$T_{fi}^{\mathrm{Trans}} = -\frac{ie^2}{2V^2}\left[\frac{1}{k^2-i\epsilon}\left\{\left(\bar{u}_{r'}(p_1')\vec{\gamma}u_r(p_1)\right)\cdot\left(\bar{u}_{s'}(p_2')\vec{\gamma}u_s(p_2)\right)-\bar{u}_{r'}(p_1')\gamma\cdot\hat{\mathbb{k}}u_r(p_1)\bar{u}_{s'}(p_2')\gamma\cdot\hat{\mathbb{k}}u_s(p_2)\right\}\right.$$

$$\left.-\frac{1}{q^2-i\epsilon}\left\{\left(\bar{u}_{r'}(p_1')\gamma u_s(p_2)\right)\cdot\left(\bar{u}_{s'}(p_2')\gamma u_r(p_1)\right)-\bar{u}_{s'}(p_2')\gamma\cdot\hat{\mathbb{q}}u_r(p_1)\bar{u}_{r'}(p_1')\gamma\cdot\hat{\mathbb{q}}u_s(p_2)\right\}\right]$$

$$\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$T_{fi}^{\mathrm{Coul}} = -\frac{ie^2}{2V^2}\left[\frac{1}{|\mathbb{k}|^2}\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)\right.$$

$$\left.-\frac{1}{|\mathbb{q}|^2}\bar{u}_{s'}(p_2')\gamma^0u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0u_s(p_2)\right]\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

ここで、$\frac{1}{|\mathbb{k}|^2}\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)$

$$=\frac{1}{k^2-i\epsilon}\cdot\frac{(k^0)^2-|\mathbb{k}|^2-i\epsilon}{|\mathbb{k}|^2}\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)$$

$$=\frac{1}{k^2-i\epsilon}\left[-\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)+\frac{1}{|\mathbb{k}|^2}\bar{u}_{r'}(p_1')\gamma^0(p_1'^0-p_1^0)u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0(-p_2'^0+p_2^0)u_s(p_2)\right]$$

$$=\frac{1}{k^2-i\epsilon}\left[-\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)+\frac{1}{|\mathbb{k}|^2}\bar{u}_{r'}(p_1')\gamma\cdot\mathbb{k}u_r(p_1)\bar{u}_{s'}(p_2')\gamma\cdot\mathbb{k}u_s(p_2)\right].$$

$$=\frac{1}{k^2-i\epsilon}\left[-\bar{u}_{r'}(p_1')\gamma^0u_r(p_1)\bar{u}_{s'}(p_2')\gamma^0u_s(p_2)+\bar{u}_{r'}(p_1')\gamma\cdot\hat{\mathbb{k}}u_r(p_1)\bar{u}_{s'}(p_2')\gamma\cdot\hat{\mathbb{k}}u_s(p_2)\right]$$

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$$\frac{1}{|\mathbf{q}|^2}\bar{u}_{s'}(p_2')\gamma^0 u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0 u_s(p_2)$$

$$=\frac{1}{q^2-i\epsilon}\cdot\frac{(q^0)^2-|\mathbf{q}|^2-i\epsilon}{|\mathbf{q}|^2}\bar{u}_{s'}(p_2')\gamma^0 u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0 u_s(p_2)$$

$$=\frac{1}{q^2-i\epsilon}\left[-\bar{u}_{s'}(p_2')\gamma^0 u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0 u_s(p_2)+\frac{1}{|\mathbf{q}|^2}\bar{u}_{s'}(p_2')\gamma^0(p_2'^0-p_1^0)u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0(-p_1'^0+p_2^0)u_s(p_2)\right]$$

$$=\frac{1}{q^2-i\epsilon}\left[-\bar{u}_{s'}(p_2')\gamma^0 u_r(p_1)\bar{u}_{r'}(p_1')\gamma^0 u_s(p_2)+\bar{u}_{s'}(p_2')\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\, u_r(p_1)\bar{u}_{r'}(p_1')\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\, u_s(p_2)\right]$$

より

$$T_{fi}^{Trans}+T_{fi}^{Coul}=-\frac{ie^2}{2V^2}\Bigg[\frac{1}{k^2-i\epsilon}(\bar{u}_{r'}(p_1')\boldsymbol{\gamma}u_r(p_1))\cdot(\bar{u}_{s'}(p_2')\boldsymbol{\gamma}u_s(p_2))$$

$$-\frac{1}{k^2-i\epsilon}\bar{u}_{r'}(p_1')\cancel{\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}\,u_r(p_1)}\,\bar{u}_{s'}(p_2')\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}\,u_s(p_2)$$

$$-\frac{1}{q^2-i\epsilon}(\bar{u}_{r'}(p_1')\boldsymbol{\gamma}u_s(p_2))\cdot(\bar{u}_{s'}(p_2')\boldsymbol{\gamma}u_r(p_1))$$

$$+\frac{1}{q^2-i\epsilon}\bar{u}_{r'}(p_1')\cancel{\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\,u_s(p_2)}\,\bar{u}_{s'}(p_2')\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\,u_r(p_1)$$

$$-\frac{1}{k^2-i\epsilon}(\bar{u}_{r'}(p_1')\gamma^0 u_r(p_1))(\bar{u}_{s'}(p_2)\gamma^0 u_s(p_2))+\frac{1}{k^2-i\epsilon}\bar{u}_{r'}(p_1')\cancel{\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}\,u_r(p_1)}\,\bar{u}_{s'}(p_2')\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}\,u_s(p_2)$$

$$+\frac{1}{q^2-i\epsilon}(\bar{u}_{s'}(p_2')\gamma^0 u_r(p_1))(\bar{u}_{r'}(p_1')\gamma^0 u_s(p_2))-\frac{1}{q^2-i\epsilon}\bar{u}_{s'}(p_2)\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\,\cancel{u_r(p_1)\bar{u}_{r'}(p_1')}\boldsymbol{\gamma}\cdot\hat{\mathbf{q}}\,u_s(p_2)\Bigg]$$

$$\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

$$=\frac{ie^2}{2V^2}\Bigg[\frac{1}{k^2-i\epsilon}\cdot\bar{u}_{r'}(p_1')\gamma^\mu u_r(p_1)\bar{u}_{s'}(p_2')\gamma_\mu u_s(p_2)$$

$$-\frac{1}{q^2-i\epsilon}\bar{u}_{s'}(p_2')\gamma^\mu u_r(p_1)\bar{u}_{r'}(p_1')\gamma_\mu u_s(p_2)\Bigg]\times(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)$$

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$$|T_{fi}|^2 = \frac{e^4}{4V^4}\Bigg[\frac{1}{(p_1'-p_1)^4}\Big|\underbrace{\bar{u}_{r'}(p_1')\gamma^\mu u_r(p_1)\bar{u}_{s'}(p_2')\gamma_\mu u_s(p_2)}_{M_A}\Big|^2 + \frac{1}{(p_2'-p_1)^4}\Big|\underbrace{\bar{u}_{s'}(p_2')\gamma^\mu u_r(p_1)\bar{u}_{r'}(p_1')\gamma_\mu u_s(p_2)}_{M_B}\Big|^2$$

$$-\frac{1}{(p_1'-p_1)^2(p_2'-p_1)^2}\underbrace{\Big[\bar{u}_{r'}(p_1')\gamma^\mu u_r(p_1)\bar{u}_{s'}(p_2')\gamma_\mu u_s(p_2)\Big]^*\bar{u}_{s'}(p_2')\gamma^\nu u_r(p_1)\bar{u}_{r'}(p_1')\gamma_\nu u_s(p_2)}_{M_C}$$

$$-\frac{1}{(p_1'-p_1)^2(p_2'-p_1)^2}\underbrace{\bar{u}_{r'}(p_1')\gamma^\mu u_r(p_1)\bar{u}_{s'}(p_2')\gamma_\mu u_s(p_2)\Big[\bar{u}_{s'}(p_2')\gamma^\nu u_r(p_1)\bar{u}_{r'}(p_1')\gamma_\nu u_s(p_2)\Big]^*}_{M_D}\Bigg]$$

$$= \qquad \times\left\{(2\pi)^4\delta^{(4)}(p_1'+p_2'-p_1-p_2)\right\}^2$$

$$\circ\ \frac{1}{4}\sum_{spin} M_A = \frac{1}{4(p_1'-p_1)^4}\sum_{spin}\bar{u}^*_{r'}(p_1')_a(\gamma^\mu)^*_{ab}u^*_r(p_1)_b\,\bar{u}^*_{s'}(p_2')_c(\gamma_\mu)^*_{cd}u^*_s(p_2)_d\,\bar{u}_{r'}(p_1')_e(\gamma^\nu)_{ef}u_r(p_1)_f\,\bar{u}_{s'}(p_2')_g(\gamma_\nu)_{gh}u_s(p_2)_h$$

$$= \frac{1}{4(p_1'-p_1)^4}\sum u^*_r(p_1)_b(\gamma^\mu)^\dagger_{ba}\bar{u}^*_{r'}(p_1')_a\,\bar{u}_{r'}(p_1')_e(\gamma^\nu)_{ef}u_r(p_1)_f\,u^*_s(p_2)_d(\gamma_\mu)^\dagger_{dc}\bar{u}^*_{s'}(p_2')_c\,\bar{u}_{s'}(p_2')_g(\gamma_\nu)_{gh}u_s(p_2)_h$$

$$= \frac{1}{4(p_1'-p_1)^4}\sum u_r(p_1)_f\bar{u}_r(p_1)_b(\gamma^\mu)_{ba}u_{r'}(p_1')_a\bar{u}_{r'}(p_1')_e(\gamma^\nu)_{ef}\,u_s(p_2)_h\bar{u}_s(p_2)_d(\gamma_\mu)_{dc}u_{s'}(p_2')_c\bar{u}_{s'}(p_2')_g(\gamma_\nu)_{gh}$$

$$= \frac{1}{4(p_1'-p_1)^4}\mathrm{Tr}\left[\frac{\not{p}_1+m}{2E_{p_1}}\gamma^\mu\frac{\not{p}_1'+m}{2E_{p_1'}}\gamma^\nu\right]\times\mathrm{Tr}\left[\frac{\not{p}_2+m}{2E_{p_2}}\gamma_\mu\frac{\not{p}_2'+m}{2E_{p_2'}}\gamma_\nu\right]$$

$$= \frac{1}{64E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}(p_1'-p_1)^4}\mathrm{Tr}\left[\not{p}_1\gamma^\mu\not{p}_1'\gamma^\nu+m^2\gamma^\mu\gamma^\nu\right]\times\mathrm{Tr}\left[\not{p}_2\gamma_\mu\not{p}_2'\gamma_\nu+m^2\gamma_\mu\gamma_\nu\right]$$

$$= \frac{4^2}{64(p_1'-p_1)^4E_{p_1'}E_{p_2'}E_{p_1}E_{p_2}}\left[p_1^\mu p_1'^\nu-(p_1\cdot p_1')g^{\mu\nu}+p_1^\nu p_1'^\mu+m^2g^{\mu\nu}\right]\left[p_{2\mu}p_{2\nu}'-(p_2\cdot p_2')g_{\mu\nu}+p_{2\nu}p_{2\mu}'+m^2g_{\mu\nu}\right]$$

$$= \frac{1}{4(p_1'-p_1)^4E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}}\Big[(p_1\cdot p_2)(p_1'\cdot p_2')-(p_1\cdot p_1')(p_2\cdot p_2')+(p_1\cdot p_2')(p_1'\cdot p_2)+m^2(p_1\cdot p_1')$$

$$-\left\{(p_1\cdot p_1')-m^2\right\}\left((p_2\cdot p_2')-4(p_2\cdot p_2')+(p_2\cdot p_2')+4m^2\right)$$

$$+(p_1\cdot p_2')(p_1'\cdot p_2)-(p_1\cdot p_1')(p_2\cdot p_2')+(p_1\cdot p_2)(p_1'\cdot p_2')+m^2(p_1\cdot p_1')\Big]$$

$$= \frac{1}{4(p_1'-p_1)^4E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}}\Big[2(p_1\cdot p_2)(p_1'\cdot p_2')+2(p_1\cdot p_2')(p_1'\cdot p_2)+2m^2\big(2m^2-(p_1\cdot p_1')-(p_2\cdot p_2')\big)\Big]$$

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M_Bは、$p_1' \leftrightarrow p_2'$ として.

$$\frac{1}{4}\sum M_B = \frac{1}{(p_2'-p_1)^4 E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}}\left[(p_1\cdot p_2)(p_1'\cdot p_2') + (p_1\cdot p_1')(p_2\cdot p_2') + m^2\left\{2m^2-(p_1\cdot p_2')-(p_2\cdot p_1')\right\}\right]$$

$$\frac{1}{4}\sum_{spin} M_C = \frac{1}{4}\sum \bar{u}_{r'}^*(p_1')_a(\gamma^\mu)^*_{ab}u_r^*(p_1)_b\,\bar{u}_s^*(p_2')_c(\gamma_\mu)^*_{cd}u_s^*(p_2)_d\,\bar{u}_{s'}(p_2')_e(\gamma^\nu)_{ef}u_r(p_1)_f\,\bar{u}_{r'}(p_1')_g(\gamma_\nu)_{gh}u_s(p_2)_h$$

$$= \frac{1}{4}\sum u_r^*(p_1)_b(\gamma^\mu)^\dagger_{ba}\bar{u}_{r'}^*(p_1')_a\,\bar{u}_{r'}(p_1')_g(\gamma_\nu)_{gh}u_s(p_2)_h\,u_s^*(p_2)_d(\gamma_\mu)^\dagger_{dc}\bar{u}_{s'}^*(p_2')_c\,\bar{u}_{s'}(p_2')_e(\gamma^\nu)_{ef}u_r(p_1)_f$$

$$= \frac{1}{4}\sum u_r(p_1)_f\bar{u}_r(p_1)_b(\gamma^\mu)_{ba}u_{r'}(p_1')_a\bar{u}_{r'}(p_1')_g(\gamma^\nu)_{gh}u_s(p_2)_h\bar{u}_s(p_2)_d(\gamma_\mu)_{dc}u_{s'}(p_2')_c\bar{u}_{s'}(p_2')_e(\gamma_\nu)_{ef}$$

$$= \frac{1}{4}\mathrm{Tr}\left[\frac{\not{p}_1+m}{2E_{p_1}}\gamma^\mu\frac{\not{p}_1'+m}{2E_{p_1'}}\gamma^\nu\frac{\not{p}_2+m}{2E_{p_2}}\gamma_\mu\frac{\not{p}_2'+m}{2E_{p_2'}}\gamma_\nu\right]$$

$$= \frac{1}{64E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}}\mathrm{Tr}\left[(\not{p}_1+m)\gamma^\mu(\not{p}_1'+m)\gamma^\nu(\not{p}_2+m)\gamma_\mu(\not{p}_2'+m)\gamma_\nu\right]$$

$$= \frac{1}{64E_{p_1}E_{p_2}E_{p_1'}E_{p_2'}}\mathrm{Tr}\Big[\underset{①}{\not{p}_1\gamma^\mu\not{p}_1'\gamma^\nu\not{p}_2\gamma_\mu\not{p}_2'\gamma_\nu} + \underset{②}{m^2\not{p}_1\gamma^\mu\not{p}_1'\gamma^\nu\gamma_\mu\gamma_\nu} + \underset{③}{m^2\not{p}_1\gamma^\mu\gamma^\nu\not{p}_2\gamma_\mu\gamma_\nu}$$

$$+\underset{④}{m^2\not{p}_1\gamma^\mu\gamma^\nu\gamma_\mu\not{p}_2'\gamma_\nu} + \underset{⑤}{m^2\gamma^\mu\not{p}_1'\gamma^\nu\not{p}_2\gamma_\mu\gamma_\nu} + \underset{⑥}{m^2\gamma^\mu\not{p}_1'\gamma^\nu\gamma_\mu\not{p}_2'\gamma_\nu}$$

$$+\underset{⑦}{m^2\gamma^\mu\gamma^\nu\not{p}_2\gamma_\mu\not{p}_2'\gamma_\nu} + \underset{⑧}{m^4\gamma^\mu\gamma^\nu\gamma_\mu\gamma_\nu}\Big]$$

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①

$$\gamma^\nu\gamma_\mu\gamma_\nu = \gamma^\nu(2g_{\mu\nu}-\gamma_\nu\gamma_\mu) = 2\gamma_\mu - 4\gamma_\mu = -2\gamma_\mu$$

$$\mathrm{Tr}\left[\not p_1 \gamma^\mu \not p_1' \gamma^\nu \not p_2 \gamma_\mu \not p_2' \gamma_\nu\right]$$

$$= \mathrm{Tr}\left[(2P_1^\mu - \gamma^\mu \not p_1)\not p_1'\gamma^\nu(2P_{2\mu} - \gamma_\mu \not p_2)\not p_2'\gamma_\nu\right]$$

$$= \mathrm{Tr}\left[4(P_1\cdot P_2)\not p_1'\gamma^\nu\not p_2'\gamma_\nu - 2P_1^\mu(P_2\cdot P_2')\not p_1'\gamma^\nu\gamma_\mu\gamma_\nu - 2P_{2\mu}(P_1\cdot P_1')\gamma^\mu\gamma^\nu\not p_2'\gamma_\nu + (P_1\cdot P_1')(P_2\cdot P_2')\gamma^\mu\gamma^\nu\gamma_\mu\gamma_\nu\right]$$

$$= \mathrm{Tr}\left[4(P_1\cdot P_2)\{(P_1'\cdot P_2') - 4(P_1'\cdot P_2') + (P_1'\cdot P_2')\} + 4P_1^\mu(P_2\cdot P_2')\not p_1'\gamma_\mu + 4P_{2\mu}(P_1\cdot P_1')\gamma_\mu\not p_2' - 2(P_1\cdot P_1')(P_2\cdot P_2')\gamma^\mu\gamma_\mu\right]$$

$$= \mathrm{Tr}\left[-8(P_1\cdot P_2)(P_1'\cdot P_2') + 4(P_1\cdot P_1')(P_2\cdot P_2') + 4(P_1\cdot P_1')(P_2\cdot P_2') - 8(P_1\cdot P_1')(P_2\cdot P_2')\right]$$

$$= -32(P_1\cdot P_2)(P_1'\cdot P_2')$$

②

$$\mathrm{Tr}\left[m^2\not p_1\gamma^\mu\not p_1'\gamma^\nu\gamma_\mu\gamma_\nu\right] = m^2\,\mathrm{Tr}\left[-2\not p_1\gamma^\mu\not p_1'\gamma_\mu\right]$$

$$= -2m^2\cdot 4\left[(P_1\cdot P_1') - 4(P_1\cdot P_1') + (P_1\cdot P_1')\right] = 16m^2(P_1\cdot P_1')$$

③

$$\mathrm{Tr}\left[m^2\not p_1\gamma^\mu\gamma^\nu\not p_2\gamma_\mu\gamma_\nu\right] = m^2\,\mathrm{Tr}\left[2P_1^\mu\gamma^\nu\not p_2\gamma_\mu\gamma_\nu - \not p_1\gamma^\nu\not p_2\gamma_\mu\gamma_\nu\gamma^\mu\right]$$

$$= m^2\,\mathrm{Tr}\left[+8P_1^\mu\not p_2\gamma_\mu + 2\not p_1\gamma^\nu\not p_2\gamma_\nu\right]$$

$$= 4m^2\left[+8(P_1\cdot P_2) + 2\{(P_1\cdot P_2) - 4(P_1\cdot P_2) + (P_1\cdot P_2)\}\right]$$

$$= +16m^2(P_1\cdot P_2)$$

④

$$\mathrm{Tr}\left[m^2\not p_1\gamma^\mu\gamma^\nu\gamma_\mu\not p_2'\gamma_\nu\right] = m^2\,\mathrm{Tr}\left[-2\not p_1\gamma^\nu\not p_2'\gamma_\nu\right]$$

$$= -8m^2\left[(P_1\cdot P_2') - 4(P_1\cdot P_2') + (P_1\cdot P_2')\right] = 16m^2(P_1\cdot P_2')$$

⑤

$$\mathrm{Tr}\left[m^2\gamma^\mu \not{p}_1'\gamma^\nu \not{p}_2 \gamma_\mu\gamma_\nu\right] = -2m^2\,\mathrm{Tr}\left[\not{p}_1'\gamma^\nu \not{p}_2\gamma_\nu\right]$$

$$= -8m^2\left[(p_1'\cdot p_2) - 4(p_1'\cdot p_2) + (p_1'\cdot p_2)\right] = 16m^2(p_1'\cdot p_2)$$

⑥

$$\mathrm{Tr}\left[m^2\gamma^\mu \not{p}_1'\gamma^\nu\gamma_\mu \not{p}_2'\gamma_\nu\right] = m^2\,\mathrm{Tr}\left[2p_1'^\mu\gamma^\nu\gamma_\mu \not{p}_2'\gamma_\nu - \not{p}_1'\gamma^\mu\gamma^\nu\gamma_\mu \not{p}_2'\gamma_\nu\right]$$

$$= m^2\,\mathrm{Tr}\left[2(p_1'\cdot p_2')\gamma^\nu\gamma_\nu + 2\not{p}_1'\gamma^\nu \not{p}_2'\gamma_\nu\right]$$

$$= 8m^2\left[4(p_1'\cdot p_2') + (p_1'\cdot p_2') - 4(p_1'\cdot p_2') + (p_1'\cdot p_2')\right]$$

$$= 16m^2(p_1'\cdot p_2')$$

⑦

$$\mathrm{Tr}\left[m^2\gamma^\mu\gamma^\nu \not{p}_2\gamma_\mu \not{p}_2'\gamma_\nu\right] = -2m^2\,\mathrm{Tr}\left[\gamma^\mu \not{p}_2\gamma_\mu \not{p}_2'\right]$$

$$= -8m^2\left[(p_2\cdot p_2') - 4(p_2\cdot p_2') + (p_2\cdot p_2')\right] = 16m^2(p_2\cdot p_2')$$

⑧

$$\mathrm{Tr}\left[m^4\gamma^\mu\gamma^\nu\gamma_\mu\gamma_\nu\right] = m^4\,\mathrm{Tr}\left[-2\gamma^\mu\gamma_\mu\right] = -32m^4$$

$$\therefore \frac{1}{4}\sum_{spin} M_C = \frac{1}{64(P_2'-P_1)^2(P_1'-P_1)^2 E_{P_1}E_{P_2}E_{P_1'}E_{P_2'}}\Big[-32(P_1\cdot P_2)(P_1'\cdot P_2') + 16m^2(P_1\cdot P_1') + 16m^2(P_1\cdot P_2) + 16m^2(P_1\cdot P_2')$$

$$+16m^2(P_1'\cdot P_2) + 16m^2(P_1'\cdot P_2') + 16m^2(P_2\cdot P_2') - 32m^4\Big]$$

$$= \frac{1}{4(P_2'-P_1)^2(P_1'-P_1)^2 E_{P_1}E_{P_2}E_{P_1'}E_{P_2'}}\Big[-2(P_1\cdot P_2)(P_1'\cdot P_2') + m^2(P_1\cdot P_1') + m^2(P_2\cdot P_2')$$

$$+ m^2\{P_1\cdot(P_2+P_2')\} + m^2\{P_1'\cdot(P_2+P_2')\} - 2m^4\Big]$$

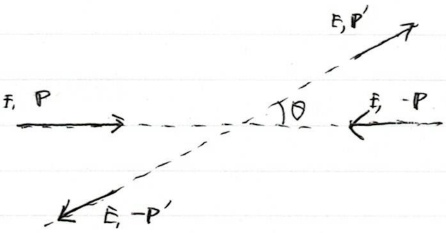
$$= \frac{+1}{4(P_2'-P_1)^2(P_1'-P_1)^2 E_{P_1}E_{P_2}E_{P_1'}E_{P_2'}}\Big[-2(P_1\cdot P_2)(P_1'\cdot P_2') + m^2(P_1+P_1')\cdot(P_2+P_2') + m^2(P_1\cdot P_1') + m^2(P_2\cdot P_2') - 2m^4\Big]$$

よって

$$\frac{1}{4}\sum_{spins} M_D = \frac{+1}{4(P_2'-P_1)^2(P_1'-P_1)^2 E_{P_1}E_{P_2}E_{P_1'}E_{P_2'}}\Big[-2(P_1\cdot P_2)(P_1'\cdot P_2') + m^2(P_1+P_2')\cdot(P_2+P_1') + m^2(P_1\cdot P_2') + m^2(P_2\cdot P_1') - 2m^4\Big]$$

$$= \frac{1}{4(P_2'-P_1)^2(P_1'-P_1)^2 E_{P_1}E_{P_2}E_{P_1'}E_{P_2'}}\Big[-2(P_1\cdot P_2)(P_1'\cdot P_2') + m^2(P_1+P_1')\cdot(P_2+P_2') + m^2(P_1\cdot P_1') + m^2(P_2\cdot P_2') - 2m^4\Big]$$

重心系をとると、 $P_1=(E,\mathbf{p})$ $P_2=(E,-\mathbf{p})$ $P_1'=(E,\mathbf{p}')$, $P_2'=(E,-\mathbf{p}')$



$$P_1\cdot P_1' = P_2\cdot P_2' = E^2 - \mathbf{p}\cdot\mathbf{p}' = E^2 - p^2\cos\theta.$$

$$P_1\cdot P_2' = P_1'\cdot P_2 = E^2 + \mathbf{p}\cdot\mathbf{p}' = E^2 + p^2\cos\theta.$$

$$P_1\cdot P_2 = P_1'\cdot P_2' = E^2 + p^2$$

$$(P_1'-P_1)^2 = P_1'^2 + P_1^2 - 2P_1\cdot P_1' = 2E^2 - 2p^2 - 2E^2 + 2p^2\cos\theta$$
$$= -2p^2(1-\cos\theta)$$

$$(P_2'-P_1)^2 = 2E^2 - 2p^2 - 2E^2 - 2p^2\cos\theta$$
$$= -2p^2(1+\cos\theta)$$

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$$|T_{fi}|^2 = \frac{e^4}{4V^4}\left[\frac{1}{8E^4P^4(1-\cos\theta)^2}\left\{(E^2+P^2)^2+(E^2+P^2\cos\theta)^2+m^2\left(2m^2-2(E^2-P^2\cos\theta)\right)\right\}\right.$$

$$+\frac{1}{8E^4P^4(1+\cos\theta)^2}\left\{(E^2+P^2)^2+(E^2-P^2\cos\theta)^2+m^2\left(2m^2-2(E^2+P^2\cos\theta)\right)\right\}$$

$$\left.-\frac{1}{8E^4P^4\sin^2\theta}\left\{-2(E^2+P^2)^2+m^2\left(2E^2+2P^2+2E^2+\cancel{2P^2\cos\theta}+2E^2-\cancel{2P^2\cos\theta}\right)-2m^4\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{(1+\cos\theta)^2}{\sin^4\theta}\left\{(E^2+P^2)^2+E^4+P^4\cos^2\theta+2E^2P^2\cos\theta+2m^2(m^2-E^2+P^2\cos\theta)\right\}\right.$$

$$+\frac{(1-\cos\theta)^2}{\sin^4\theta}\left\{(E^2+P^2)^2+E^4+P^4\cos^2\theta-2E^2P^2\cos\theta+2m^2(m^2-E^2-P^2\cos\theta)\right\}$$

$$\left.+\frac{2}{\sin^2\theta}\left\{(E^2+P^2)^2-m^2(3E^2+P^2)+m^4\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{1+\cos^2\theta}{\sin^4\theta}\left\{2(E^2+P^2)^2+2E^4+2P^4\cos^2\theta+4m^2(m^2-E^2)\right\}\right.$$

$$\left.+\frac{2\cos\theta}{\sin^4\theta}\left\{4E^2P^2\cos\theta+4m^2P^2\cos\theta\right\}+\frac{2}{\sin^2\theta}\left\{(E^2+P^2)^2-(E^2-P^2)(3E^2+P^2)+(E^2-P^2)^2\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{2-\sin^2\theta}{\sin^4\theta}\left\{2(E^2+P^2)^2+2E^4+2P^4-2P^4\sin^2\theta-4m^2P^2\right\}\right.$$

$$\left.+\frac{2-2\sin^2\theta}{\sin^4\theta}\left\{4E^2P^2+4m^2P^2\right\}+\frac{2}{\sin^2\theta}\left\{(E^2+P^2)^2-(E^2-P^2)(2E^2+2P^2)\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{2}{\sin^4\theta}\left\{2(E^2+P^2)^2+2E^4+2P^4-\cancel{4m^2P^2}+4E^2P^2+\cancel{4m^2P^2}\right\}+2P^4\right.$$

$$\left.-\frac{2}{\sin^2\theta}\left\{\cancel{(E^2+P^2)^2}+E^2+P^4-2m^2P^2+2P^4+4E^2P^2+4m^2P^2-\cancel{(E^2+P^2)^2}+(E^2-P^2)(2E^2+2P^2)\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{2}{\sin^4\theta}\left\{2(E^2+P^2)^2+2(E^2+P^2)^2\right\}+2P^4\right.$$

$$\left.-\frac{2}{\sin^2\theta}\left\{E^2+3P^4+2m^2P^2+4E^2P^2+2E^4-2P^4\right\}\right]$$

$$=\frac{e^4}{32V^4E^4P^4}\left[\frac{8(E^2+P^2)^2}{\sin^4\theta}+2P^4-\frac{2}{\sin^2\theta}\left\{3E^4+P^4+2(E^2-P^2)P^2+4E^2P^2\right\}\right]$$

$$= \frac{e^4}{32V^4E^4p^4}\left[\frac{8(E^2+p^2)^2}{\sin^4\theta} + 2p^4 - \frac{2}{\sin^2\theta}\left\{3E^4 + p^4 + 2E^2p^2 - 2p^4 + 4E^2p^2\right\}\right]$$

$$= \frac{e^4}{32V^4E^4p^4}\left[\frac{8(E^2+p^2)^2}{\sin^4\theta} + 2p^4 - \frac{2}{\sin^2\theta}\left\{3E^4 + 6E^2p^2 - p^4\right\}\right]$$

$$= \frac{e^4}{32V^4E^4p^4}\left[\frac{8(E^2+p^2)^2}{\sin^4\theta} + 2p^4 - \frac{2}{\sin^2\theta}\left\{3(E^2+p^2)^2 - 4p^4\right\}\right]$$

$$= \frac{e^4}{32V^4E^4p^4}\left[\frac{8(E^2+p^2)^2}{\sin^4\theta} - \frac{6(E^2+p^2)^2}{\sin^2\theta} + 2p^4 + \frac{8p^4}{\sin^2\theta}\right]$$

$$= \frac{e^4}{16V^4E^4p^4\sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2\sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right]$$

相対速度について、重心系で

$$v_{rel} E_{P_1} E_{P_2} = \sqrt{(P_1 \cdot P_2)^2 - m^4} = \sqrt{(E^2+p^2)^2 - m^4} = \sqrt{(2p^2+m^2)^2 - m^4}$$

$$= \sqrt{4p^4 + 4p^2 m^2} = \sqrt{4p^2(p^2+m^2)} = 2pE$$

よって、全断面積は

$$\sigma = V\int \frac{d^3 p_1'}{(2\pi)^3} \cdot V\int \frac{d^3 p_2'}{(2\pi)^3} \cdot \frac{V}{v_{rel}} \cdot \frac{1}{T} \frac{e^4}{16 V^4 E^4 p^4 \sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2 \sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right]$$

$$\times (2\pi)^4 TV \cdot \delta^{(4)}(P_1' + P_2' - P_1 - P_2)$$

$$= \int d^3 p_1' \frac{e^4}{(2\pi)^2 \cdot 16 v_{rel} E^4 p^4 \sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2 \sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right] \delta(E_{P_1'} + E_{P_2'} - E_{P_1} - E_{P_2})$$

これより微分散乱断面積は

$$\left(\frac{d\sigma}{d\Omega}\right)_{COM} = \int_0^\infty d|\mathbf{p}_1'|\, |\mathbf{p}_1'|^2 \frac{\alpha^2}{4 \cdot 2 E^3 p^5 \sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2 \sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right] \delta(E_{P_1'} + E_{P_2'} - E_{P_1} - E_{P_2})$$

$$= \int_m^\infty dE_{P_1'} \frac{\alpha^2}{8E^2 p^4 \sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2 \sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right] \delta(E_{P_1'} + E_{P_2'} - E_{P_1} - E_{P_2})$$

$$= \frac{\alpha^2}{8E^2 p^4 \sin^4\theta}\left[4(E^2+p^2)^2 - 3(E^2+p^2)^2 \sin^2\theta + p^4(\sin^4\theta + 4\sin^2\theta)\right]$$